ECE 109 - NODE ANALYSIS - INVESTIGATION 15
NODE EQUATIONS - A GUIDED TOUR

FALL 2006 **A.P. FELZER**

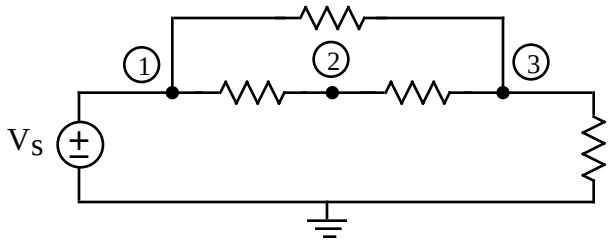
To do "well" on this investigation you must not only get the right answers but must also do neat, complete and concise writeups that make obvious what each problem is, how you're solving the problem and what your answer is. You also need to include drawings of all circuits as well as appropriate graphs and tables.

We showed in the last investigation how we can calculate all the circuit element voltages and currents in resistor circuits from the node voltages as follows

(1) First calculate the voltage drops across the resistors by subtracting the corresponding node voltages
(2) Then make use of the resistor voltages to calculate the resistor currents
(3) And finally make use of the resistor currents to calculate the voltage source currents

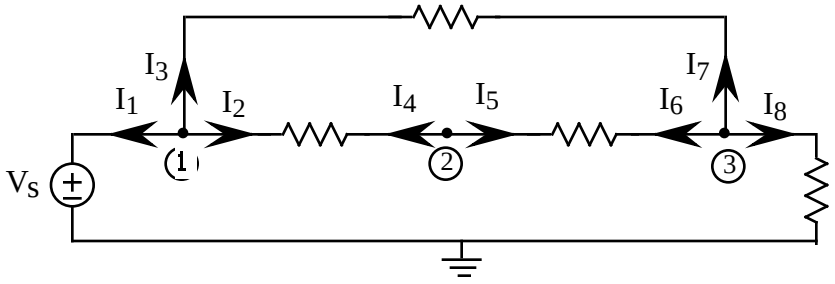
Be sure to **memorize** this result if you haven't already.

The objective of this Investigation is to show how to use the following algorithm to calculate the node voltages of resistor circuits like the following



with all the voltage sources connected to a common reference -

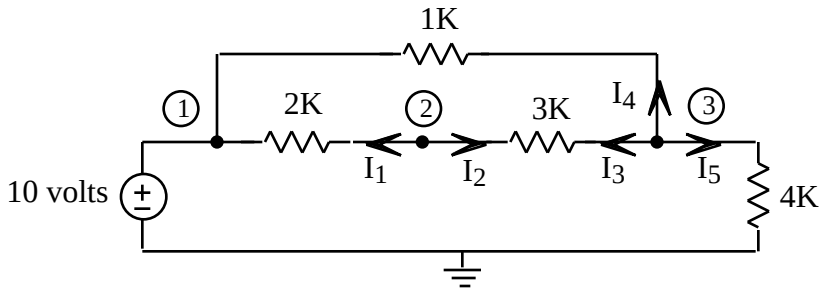
(1) Choose a reference node - usually at the negative terminal of the voltage source as we've done for this circuit
(2) Write KCL at each of the nodes - except those with voltage sources connected to the reference - in terms of the currents I_1, I_2, $\cdots$ flowing through the resistors and current sources as follows



(3) Express the resistor currents in part (2) in terms of the node voltages $V_1 = V_S$, V_2 and V_3
(4) Put your equations in matrix form
(5) Solve for the unknown node voltages either by hand, calculator or computer
(6) Make use of the node voltages to calculate the currents through and voltages across the elements

Be sure to take a look at the **Computer Demos** on Node Equations.

1. Carry out the steps of the algorithm above to find the node voltages of the following circuit



In particular

a. Choose a reference node - already done
b. Write KCL equations at nodes 2 and 3 - the nodes that don't have a voltage source connected to the reference. In particular, write the KCL equation at node 2 for currents I_1 and I_2 and the KCL equation at node 3 for currents I_3, I_4 and I_5. *Be sure to label which equation is for which node* as follows

Node	KCL Equation
1	. . . .

c. Make use of Ohm's Law to express the resistor currents in your KCL equations in terms of the node voltages. Again be sure to label your equations as follows

Node	Equation
1	. . . .

d. Put your equations in **matrix** form Note that we put a set of linear equations like these

$$2V_1 + 3V_2 = 5V_S$$
$$4V_1 - 2V_2 = 0$$

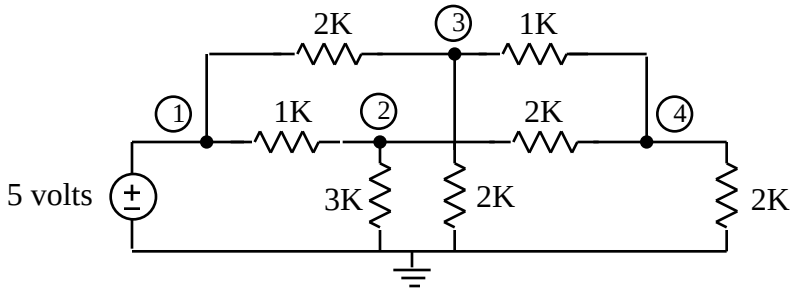
in matrix form by writing them as follows

$$\begin{pmatrix} 2 & 3 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 5V_s \\ 0 \end{pmatrix}$$

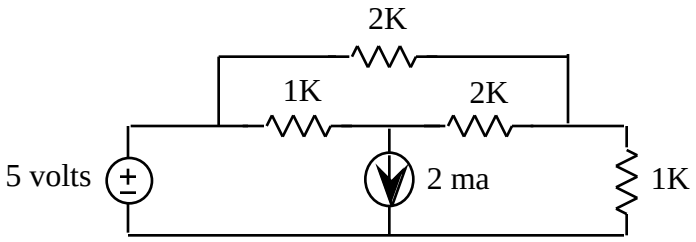
e. Solve for the unknown node voltages either by hand, calculator or computer. Do you consider your results reasonable. Explain

Make sure to note that this method of analyzing general resistor circuits is simply a generalization on our method for analyzing parallel circuits

2. Write and put in matrix form the node equations of the following circuit. Note that we put a dot whenever two crossing wires are connected. Note that if no dot is drawn then crossing wires are not connected

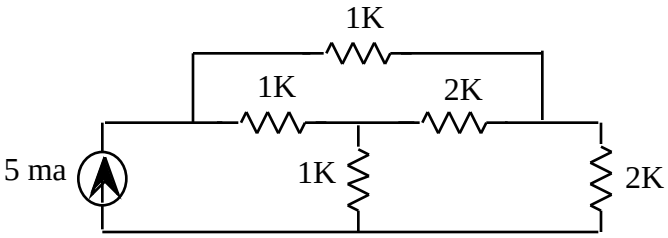


3. Write and put in matrix form the node equations of the following circuit

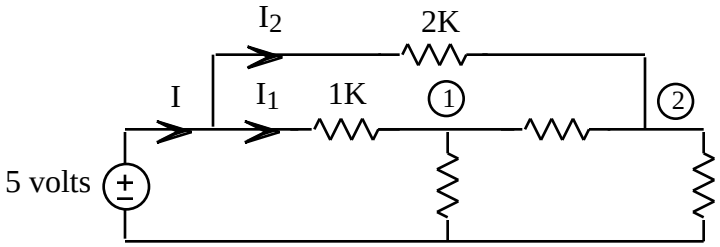


4. Write and put in matrix form the node equations of a circuit that you make up

5. Write and put in matrix form the node equations of the following circuit



6. Given the following circuit with $V_1 = 3$ volts and $V_2 = 2$ volts



   a. Find I_1 and I_2
   b. Find I

7. Explain why we always end up with the same number of equations as unknowns when we write node equations

8. Math Review - Given a point *P* in the xy-plane with coordinates (-2, 3)
   a. Sketch *P* in the plane
   b. How far is *P* from the origin